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An Inventory Model For Deteriorating Items With Selling Price Dependent Demand And Variable Deterioration Under Inflation With Time Dependent Holding Cost

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Abstract— In this, we considered an inventory model in which demand is taken as a function of selling price and variable rate of deterioration is taken as a linear function of time and a storage time dependent holding cost. The holding cost per unit of the item is considered to be a linear function of time spent in storage. In this paper effect of inflation rate is considered and shortages are not allowed. An inventory model is developed for obtaining optimum cycle length for the given cost structure. The proposed model reduces to well known result, by choosing appropriate value of the parameters.

Index Terms— Inventory, optimal, inflation, deterioration, demand, holding cost, time
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1 INTRODUCTION

The well known square root formula is $Q=\sqrt{2C_3D/C_1}$ for economic order quantity of the item, where C_1, C_3 and D are holding cost, replenishment cost and demand rate respectively. In this formula demand rate is constant and items in inventory do not undergo deterioration. However, in real life situation, inventory loss may be due to deterioration and demand. Demand may depend on selling price, for example fruits, vegetables and consumer-goods type items. During past few years many authors have studied inventory models for deterioration items considering different demand and deterioration rate.

Selling price is one of the decisive factors in selecting a them for use. It is well known that lesser the selling price of an item, increase the demand of that item, where as higher the selling price has reverse effect. Some authors like Cohen (1), Mukherjee (4), Gupta and Jawhari (2), Kumar and Sharma (3) etc. developed inventory models taking demand as a function of selling price. Yadav.et.al(7) has developed a deterministic inventory model for deteriorating items in which demand is taken as a function of selling price and variable rate of deterioration is taken as a linear function of time and a storage time dependent holding cost.

In this article, an inventory model is developed considering demand is a function of selling price. Inflation is considered and rate of deterioration is taken as a linear function of time with time dependent holding cost. The model is solved by minimizing the total average cost. Shortages are not allowed. As a special case this model reduce to well known result.

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2 ASSUMPTIONS AND NOTATIONS

The proposed model is developed under the following assumptions and notations.

- The demand for the item is partially constant and partially selling price dependent and is assumed as
  D(p(t)) = a – b p(t) where 'a' is fixed demand; a, b>0 and a>>b.
  p(t) is the selling price of the item at time t and is taken as $p(t) = pe^{rt}$ is the selling price per unit at time 't' and 'p' is the selling price of the item at time t=0
- 'r' is the inflation rate is constant.
- Shortages are not allowed and lead time is zero.
- There is no repair or replacement of the deteriorated units during the cycle time under consideration.
- I(t) is the level of inventory at any instant of time.
- C is the unit purchase cost, k is the ordering cost per order.
- T is the cycle time.
- A variable fraction θ(t) of on hand inventory deteriorates per unit time. In the present model, the function θ(t) is assumed of the form
  $\theta(t) = \alpha + \beta t$, $0<\beta<<1$, $t>0$, $\alpha\geq 0$, $\beta\geq 0$, t = time
  - h(t) holding cost of the item at time t
    $h(t) = \delta + \Upsilon t$ $0<<\delta, \Upsilon<<1$

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3 FOR THE SYSTEM MATHEMATICAL ANALYSIS

Let I(t) be the inventory level at any time 't'. The inventory level decreases mainly due to demand and partly due to deterioration of units. The differential equation governing the system in the interval (0, T) is given by

$$\frac{dI(t)}{dt} = -\theta(t)I(t) - D(p(t)) \qquad (0 \le t \le T) \qquad (1)$$

$$= -(\alpha + \beta t)I(t) - (a - bpe^{rt}) \qquad (2)$$

Solution of the differential equation after adjusting constant of integration and initial condition $t = 0, I(t) = I(0)$

$$I(t) = \exp.\left[-\left(\alpha t + \frac{\beta t^2}{2}\right)\right]\left[-a\left(t + \frac{\alpha t^2}{2} + \frac{\beta t^2}{6}\right) + bp\left\{t + \frac{(\alpha + r)t^2}{2} + \frac{\beta t^3}{6}\right\} + I(0)\right] \qquad (3)$$

Inventory without decay $I_w(t)$ at time 't' is given by

$$\frac{d}{dt}I_w(t) = -(a - bpe^{rt})$$

$$\Rightarrow I_w(t) = -at + \frac{bpe^{rt}}{r} + I(0) - \frac{bp}{r} \qquad (4)$$

(using initial condition at $t = 0, I(t) = I(0)$)

Stock loss due to decay Z(t) at time t is given by

$$Z(t) = I_w(t) - I(t)$$

$$= -at - \frac{bp}{r}(1 - e^{rt}) + I(0) - I(t) \qquad (5)$$

equation (3) gives

$$I(0) = I(t)\exp.\left(\alpha t + \frac{\beta}{2}t^2\right) + a\left(t + \frac{\alpha}{2}t^2 + \frac{\beta}{6}t^3\right) - bp\left(t + \frac{\alpha + r}{2}t^2 + \frac{\beta}{3}t^3\right) \qquad (6)$$

Substituting value of $I(0)$ from (6) in (5), we get

$$Z(t) = -at - \frac{bp}{r}(1 - e^{rt}) - bp\left(t + \frac{\alpha + r}{2}t^2 + \frac{\beta}{3}t^3\right) + a\left(t + \frac{\alpha}{2}t^2 + \frac{\beta}{6}t^3\right) + I(t)\left[\exp.\left(\alpha t + \frac{\beta}{2}t\right) - 1\right]$$

(7)

At t=T, we get

$$Z(t) = -aT - \frac{bp}{r}(1 - e^{rt}) - bp\left(T + \frac{\alpha + r}{2}T^2 + \frac{\beta}{3}T^3\right) + a\left(T + \frac{\alpha}{2}T^2 + \frac{\beta}{6}T^3\right) \qquad (8)$$

Note that $I(T) = 0$

Order quantity is given by

$$Q_T = Z(T) + \int_0^T (a - bpe^{rt})dt$$

$$= a\left(T + \frac{\alpha}{2}T^2 + \frac{\beta}{6}T^3\right) - bp\left(T + \frac{\alpha + r}{2}T^2 + \frac{\beta}{3}T^3\right) \qquad (9)$$

Also $I(0) = Q_T$ implies

$$I(t) = \exp.\left[-\left(\alpha t + \frac{\beta}{2}t^2\right)\right]\left[a\left\{(T - t) + \frac{\alpha}{2}(T^2 - t^2) + \frac{\beta}{6}(T^3 - t^3)\right\} - bp\left\{(T - t) + \frac{\alpha + r}{2}(T^2 - t^2) + \frac{\beta}{3}(T^3 - t^3)\right\}\right]$$

(10)

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As stated earlier, the holding cost is assumed to be a linear function of time i.e

$$h(t) = \delta + \gamma t$$

In this case the C(T,p) total inventory cost per unit time can be expressed as

$$C(T,p) = \frac{k}{T} + \frac{CQ_T}{T} + \frac{1}{T}\int_0^T h(t)I(t)dt \qquad (11)$$

$$= \frac{k}{T} + C\left[\left\{a\left(1+\frac{\alpha}{2}T+\frac{\beta}{3}T^2\right)\right\} - bp\left\{1+\frac{(\alpha+r)}{2}T+\frac{\beta}{3}T^2\right\}\right] + \frac{\delta(a-bp)}{2}T + \frac{(\delta\alpha+\gamma)(a-bp)}{6}T^2$$

$$-\frac{bpr\delta}{3}T^2 + \frac{(\gamma\alpha-\beta\delta)(a-bp)}{24}T^3 - \frac{bpr\gamma}{8}T^3 + \frac{a\alpha^2\delta}{3}T^3 - \frac{b\alpha pr}{8}T^3 - \frac{\alpha^2 bp}{6}T^3$$

For minimum total average cost, the necessary criterion is

$$\frac{d}{dt}\left[C(T,p)\right] = 0 \qquad (12)$$

For fixed 'p'

$$\Rightarrow -k + T^2C\left[a\left(\frac{\alpha}{2}+\frac{2\beta}{3}T\right) - bp\left(\frac{\alpha+r}{2}+\frac{2\beta}{3}T\right)\right] + \delta(a-bp)T^2 + \frac{(\delta\alpha+\gamma)(a-bp)}{3}T^3 - \frac{2bpr\delta}{3}T^3$$

$$\frac{(\gamma\alpha-\beta\delta)(a-bp)}{8}T^4 - \frac{3bpr\gamma}{8}T^4 + a\alpha^2\delta T^4 - \frac{3bp\alpha r}{8}T^4 - \frac{\alpha^2 bp}{2}T^4 = 0$$

which can be solved for T_p numerically by using theory of equations:

Also

$$\frac{d^2}{dT^2}\left[C(T,p)\right] = C\left[a\left\{\alpha T + 2\beta T^2\right\} - bp\left\{(\alpha+r)T + 2\beta T^2\right\}\right] + 2\delta(a-bp)T + (\delta\alpha+r)(a-bp)T^2 - 2bpr\delta T^2$$

$$+\frac{(\gamma\alpha-\beta\delta)(a-bp)}{2}T^3 - \frac{3bpr(\gamma+\alpha)}{2}T^3 + 4a\alpha^2\delta T^3 - 2\alpha^2 bpT^3$$

$$\frac{d^2}{dT^2}\left[C(T,p)\right] > 0 \qquad (13)$$

<u>SPECIAL CASE:-</u>

If a=R, b=0, α=0, β=0, □=0, δ=h. Then

$$C(T) = \frac{k}{T} + CR + \frac{hRT}{2}$$

which is the standard result for non-decaying inventory.

4 Conclusion

In this paper, we have proposed inventory model for deteriorating items having selling price dependent demand and time dependent deterioration and time dependent holding cost. Shortages are not allowed and constant inflation is also considered. The model is solved by cost minimizing criterion. As a special case this model reduces to standard result for non-decaying inventory.

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